Parametric Correlation of Very Short-Time, Stress-Rupture Data for Design Purposes

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Manson-Haferd, Goldhoff-Sherby, Larson-Miller, Sherby-Dorn, and Manson-Succop time-temperature parameters are demonstrated to be successfully correlated for design purposes by computer program results obtained with 30 data sets for 20 alloys, involving 1215 rupture times from 1 to 5720 sec. Superior correlations result from use of the first two parameters. The Conrad and Korchynsky formulations are not recommended. Very short-time data are as successfully correlated as are conventional long-time data, so designers may interpolate to other temperatures, stress levels, or time ranges with accustomed degrees of accuracy.

Nomenclature

A = Manson-Succop materials constant
B = Conrad materials constant
b = Korchynsky temperature dependent term
C = Larson-Miller materials constant
c = polynomial constant
D = Korchynsky materials constant
e = natural logarithm base
E = Korchynsky materials constant
ΔH = Sherby-Dorn activation energy, materials constant
ΔH^* = Conrad activation energy, materials constant
K = temperature, °K
k = number of parameter constants
k = $m+2$ for Larson-Miller, Sherby-Dorn, Manson-Succop, Conrad, and Korchynsky parameters
k = $m+3$ for Manson-Haferd and Goldhoff-Sherby parameters
m = degree of polynomial
P = time-temperature parameter
R = universal gas constant
T = temperature, °F
T_a = Manson-Haferd materials constant
T_b = Goldhoff-Sherby materials constant
t_a = Manson-Haferd materials constant
t_b = Goldhoff-Sherby materials constant
t_r = rupture-time, hr
σ = stress, psi
σ_0 = Conrad temperature dependent term

Introduction

TEST results of creep-rupture test programs are usually presented on summary plots with log σ as the ordinate against log t_r, and with many alloys there are tendencies for lines connecting isothermal data points to be smooth curves which may approach linearity over segmental lengths. Unfortunately, with such a collection of isothermal lines, it is difficult to interpolate to conditions not exactly covered by the test programs. To help overcome these and other problems the concepts of time-temperature parameters and their master curves have been developed. The initial and most often-used approach is to combine temperature and t_r into one parameter which is plotted as the abscissa against log σ: a good parameter will result in the correlation of all data points about the master curve. Other parametric approaches have been developed, however; the engineering importance of time-temperature parameters can hardly be overstated and the pertinent technical literature is voluminous. The parameters actually have several uses, as a) to allow the designer to interpolate to temperatures, stress levels, and/or time ranges other than those involved in the experimental test program, b) to present much data in a compact form which can be used for analytical representation, c) to permit simple comparisons of different materials to display relative merits, and d) to minimize the extent of creep-rupture evaluation programs.

Conventional long-time, creep-rupture data are applicable to relatively steady-state long-life conditions as are encountered in steam and gas-turbine power plants, and rupture times in the appropriate ranges from 100 to 100,000 hr are commonly determined. However, there are many modern applications involving the start-up, ascent and re-entry of missiles, rockets, space probes, vehicles, etc., for which components must function for only very short times, as several seconds or minutes, under conditions approaching thermal shock and often involving exceptionally high stresses and temperatures. To provide appropriate test data for the designer, and to evaluate alloys, new very short-time testing techniques have been developed and have been extensively applied.

This paper shows that time-temperature parameters may be successfully applied for the correlation of very short-time, stress-rupture data and extends evaluations of the relative abilities of seven selectively chosen parameters to correlate this data. Statistical methods of achieving and evaluating results, involving highly computerized programs, are utilized to ensure that there is no ambiguity. This work compares the degrees of correlation with those of another program[1] dealing with conventional long-time t_r characteristics of nickel-base superalloys. Optimum correlations of 1215 very short-time, stress-rupture test results contained in 30 data sets obtained with 20 different alloys are presented. Given here are the first extensive evaluations of the seven considered time-temperature parameters for very short-time, stress-rupture data.

Time-Temperature Parameters

Larson and Miller's[2] 1952 breakthrough set the trend and resulted in the formulation

$$P = (T + 460)(\log t_r + C) \tag{1}$$

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The Manson-Haferd[3] linear parameter appeared almost simultaneously

$$P = (\log t_r - \log t_a)/(T - T_a) \tag{2}$$

The Sherby-Dorn[4] formulation soon followed

$$P = t_r e^{(-\Delta H/RK)} \tag{3}$$

To construct the Sherby-Dorn master curve log P is plotted as the abscissa. The Manson-Succop[5] modified linear parameter appeared in 1956

$$P = \log t_r + AT \tag{4}$$

The Goldhoff-Sherby term is quite new, and has had only two evaluations with conventional long-time creep-rupture results[1,6]

$$P = (\log t_r - \log t_b)/(1/T - 1/T_b) \tag{5}$$

Only the Larson-Miller and Sherby-Dorn methods are based on phenomenological concepts associated with rate processes; the other three approaches are completely empirical. Mathematical and graphical analyses will indicate the fundamental differences between the five parameters, will allow approximate evaluations of the various constant terms, will give significance to the various constants, and will also show that the Goldhoff-Sherby method is a modified Larson-Miller parameter.

A dislocation model for high-temperature steady-state creep is the basis for Conrad's[7,8] correlation theory

$$t_r = B/e^{-(\sigma/\sigma_0)}e^{(\Delta H^*/RT)} \tag{6}$$

The master curve is constructed by plotting $(\sigma/2.3\sigma_0)$ as the ordinate against $(-\log t_r + \Delta H^*/2.3RK)$. Although Korchynsky considered Eq. (6) as a basis for his correlation parameter, he[9] ultimately utilized another dislocation model formulation

$$t_r = D\sigma^b e^{(E/K)} \tag{7}$$

The Korchynsky master curve is plotted with $(-b \log \sigma)$ as the ordinate against $(-\log t_r + E/2.3\text{K})$. The Conrad and Korchynsky parameters are fundamentally different from the other five in that they both involve temperature dependent terms (σ_0, b), the ordinates of the resulting master curves are meant to be linear with slopes of one.

Computer Programs

Computer programs are written in Fortran V for utilization with a Univac 1108 unit. The programs for the five conventional parameters are advanced modifications of Manson's[10,11] pioneer work as expanded[6] and as further modified.[1] Orthogonal polynomial representation of master curves and linear transformations of stresses are used. Parameters are expressed by Manson's general creep-rupture formulation. Whereas representing master curves by third-order polynomials

$$P = c_0 + c_1(\log \sigma) + c_2(\log \sigma)^2 + c_3(\log \sigma)^3 \tag{8}$$

The computer selects optimum values of the constant terms in Eqs. (1–5) to minimize the standard deviation of regression

$$\{\sum_1^n [\log t_r(\text{observed}) - \log t_r(\text{predicted})]^2/n - k\}^{1/2} \tag{9}$$

Third-order polynomial approximations to master curves were selected as a result of previous experience[1]. In Eq. (9) t_r (observed) is the actual experimentally determined rupture time and t_r (predicted) is the equivalent rupture time on the polynomial approximated master curve at the same temperature and stress, so Eq. (9) really represents a measure of the scatter of all of the actual data points about the master curve. For each of the laboratory-determined stress-rupture test results the computer prints out values of a Predicted stress—the value of stress on the polynomial approximated master curve at the real values of temperature and rupture-time; b) predicted rupture-time, as just described; c) percentage deviation of stress

$$\{[\text{stress (predicted)} - \text{stress (observed)}]/\text{stress (observed)}\}100 \tag{10}$$

a) percentage deviation of rupture-time,

$$\{[t_r(\text{predicted}) - t_r(\text{observed})]/t_r(\text{observed})\}100 \tag{11}$$

For each complete data set, for each alloy, the computer also prints out values of a) average deviation of stress,

$$[\sum_1^n (\text{percentage deviation of stress})]/n \tag{12}$$

b) average deviation of rupture-time

$$[\sum_1^n (\text{percentage deviation of rupture-time})]/n \tag{13}$$

Work with the Conrad and Korchynsky parameters demanded the preparation of original computer programs, but these retain several features of the modified Manson programs. Of course, since the two correlation plots are meant to be linear with slopes of one, the third-order polynomial of Eq. (8) must be replaced by a first-order polynomial. The basic Conrad and Korchynsky procedures have been somewhat modified to allow optimum evaluations of constants and temperature-dependent terms. These modifications and the original computer programs will be described in other publications to be written after further evaluations of conventional long-time, creep-rupture data have been completed.

Alloy and Testing Information

Alloys pertinent to this paper are given indentinumbers and are referred to by standard trade designations in the first two vertical columns of Table 1. Data sets have been subdivided into three horizontal groupings. The first group of six is for alloy steels and includes one ferritic stainless steel. The second grouping of 13 data sets is for austenitic stainless steels, and it will be noted that attempts have been made to take some advantage of strengthening by cold work. The superalloys of the third group include two iron- or mixed-base alloys and eight data sets for nickel-base alloys—five of these are precipitation hardenable and one (Haynes 8202) is an experimental binary with an unusually high-tungsten content of 46.5 wt %. One cobalt-base alloy has been included in the superalloy grouping. The metallurgical conditions of the alloys are indicated by the third column.

Some pertinent testing program information for alloys evaluated by very short-time, stress-rupture techniques and included in this paper is summarized in Table 2. Listed under the multiple columns "Temperatures, °F" are the number of different test temperatures utilized in each individual testing program and the minimum and maximum test temperatures. The "Tests" column gives the number of individual test results included in each data set, and the "t_r, seconds" multiple columns give the maximum and minimum rupture times: extremes of stresses utilized in the testing programs are listed in the indicated columns. The majority of alloys were tested as full-thickness sheet specimens, but data sets 1, 2, 4, 9, and 13–17 were obtained with ¼-in.-round specimens prepared from plate or rod. Literature references listed in the second column of Table 3 have been supplemented by intimate personal contact with the testing programs or by contact and correspondence with the authors of the reports and papers.

Study of the information given in Tables 1 and 2 will show that a different grouping of the data sets would have been possible. Korchynsky's[14] test results, obtained in a test unit

Table 1 Computer-evaluated constants for the seven time-temperature parameters

No.	Alloy	Condition	L-M	M-H		S-D	G-S		M-S	C	K
			C	log t_a	T_a	ΔH	log t_b	$1/T_b$	A	ΔH*	E
1	Cor Ten	70,000 psi UTS	15.40	1.55	700	57,590	2.78	.00133	.007	68,580	398,700
2	Cor Ten	81,000 psi UTS	17.10	5.78	300	66,570	82.79	-.00500	.008	169,790	998,810
3	4130	Annealed	11.94	3.91	1000	75,430	10.46	.00100	.004	53,580	107,960
4	4130	Normalized	15.63	22.15	-2000	59,050	-11.94	.00000	.007	106,340	658,900
5	Halcomb 218	Annealed	12.38	1.48	1600	77,660	10.73	.00100	.004	41,000	-24,130
6	446	Annealed	15.56	-6.71	3000	90,780	-5.03	.00033	.006	43,110	156,130
7	301	50 % CW	13.37	9.67	2000	36,690	-5.83	.00050	.008	28,080	220,700
8	301	80 % CW	21.11	6.91	400	76,610	41.39	.00400	.013	123,510	772,760
9	304	Annealed	15.65	4.24	600	75,090	30.71	.00250	.006	83,660	489,350
10	304	Annealed	20.96	20.18	5000	138,590	-10.05	.00020	.007	85,200	254,910
11	304	5 % CW	17.90	6.63	1000	116,050	110.59	.00400	.006	70,970	321,460
12	304	10 % CW	17.41	2.42	1600	111,180	15.41	.00100	.006	69,640	350,550
13	321	Annealed	20.23	9.05	300	96,780	-73.81	-.00250	.008	13,070	90,980
14	347	Annealed	16.42	3.43	1000	87,390	15.90	.00133	.006	40,090	184,850
15	18-18-2	Annealed	17.61	22.69	-2000	90,370	-14.63	.00000	.006	35,500	208,460
16	17-7 PH	Aged, TH 1050	25.53	4.82	600	91,480	9.02	.00167	.016	127,510	777,860
17	17-7 PH	Aged, TH 1050	15.63	11.88	-600	68,960	-14.29	.00010	.006	55,520	336,190
18	17-7 PH	Aged, TH 1050	23.30	25.30	-750	80,960	-12.49	.00025	.015	62,110	380,650
19	17-7 PH	Aged, CH 900	23.04	20.99	-600	78,210	-23.83	-.00050	.014	414,820	2,534,270
20	A 286	Aged, 1350 °F	18.28	-18.75	5000	117,940	-8.94	.00020	.006	6,730	32,030
21	Multimet	Annealed	16.91	5.61	900	96,800	18.77	.00133	.007	81,670	478,910
22	Hastelloy B	Annealed	15.34	-7.36	3500	100,140	-7.46	.00020	.005	113,670	307,400
23	Hastelloy C	Annealed	17.49	6.42	1000	111,450	15.33	.00100	.006	74,350	272,950
24	Hastelloy R	Annealed	22.48	4.12	1600	148,530	21.46	.00100	.008	86,490	336,590
25	Rene 41	Annealed	20.46	-3.16	2500	131,390	-2.87	.00040	.007	94,310	384,410
26	Rene 41	Aged, 1400 °F	23.74	4.49	1300	140,690	15.01	.00100	.010	46,310	327,380
27	Inconel 702	Aged, 1350 °F	20.37	-12.95	4000	134,290	-6.09	.00030	.007	10,870	140,364
28	Nimonic 90	Aged, 1350 °F	29.87	90.92	-5000	144,770	-13.06	.00030	.014	94,270	627,610
29	Haynes 8202	Annealed	14.26	8.17	4000	97,640	-5.32	.00025	.005	73,070	263,300
30	Haynes 25	Annealed	15.59	4.29	1300	103,290	15.15	.00100	.005	25,800	105.150

Table 2 Values of the standard deviation of regression

	Temperatures, °F				t_r,seconds		Stresses, psi		Standard Deviation of Regression						
No.	No.	Min.	Max.	Tests	Min.	Max.	Min.	Max.	L-M	M-H	S-D	G-S	M-S	C	K
1	7	900	1600	69	1	103	7,500	59,000	0.383	0.353	0.380	0.366	0.386	0.351	0.345
2	8	900	1600	76	1	100	9,300	63,500	0.320	0.309	0.302	0.302	0.337	0.534	0.578
3*	5	2138	2462	19	30	1100	500	4,000	0.129	0.107	0.111	0.120	0.107	0.180	0.158
4	6	1000	1600	61	1	106	10,200	73,000	0.342	0.352	0.345	0.340	0.350	0.276	0.292
5*	3	2228	2426	12	14	924	1,000	4,000	0.159	0.150	0.153	0.162	0.151	0.115	0.147
6*	7	1814	2282	29	27	2846	500	3,000	0.116	0.073	0.138	0.078	0.105	0.124	0.057
7	3	1000	1400	16	18	1784	20,000	150,000	0.394	0.348	0.442	0.356	0.392	0.277	0.352
8	3	800	1200	12	16	1843	8,000	210,000	0.254	0.269	0.239	0.241	0.280	0.519	0.554
9	9	1200	2000	142	1	96	4,700	49,000	0.304	0.295	0.289	0.281	0.324	0.336	0.353
10*	8	2084	2516	62	9	5152	800	6,000	0.111	0.105	0.120	0.110	0.103	0.170	0.145
11*	4	2084	2444	11	83	746	1,000	5,000	0.041	0.034	0.036	0.039	0.045	0.053	0.082
12*	4	2084	2444	11	53	487	1,000	5,000	0.083	0.076	0.073	0.073	0.090	0.085	0.099
13	8	1300	2000	110	1	97	6,000	47,500	0.245	0.243	0.233	0.231	0.257	0.415	0.965
14	5	1595	2300	34	3	5361	900	28,000	0.174	0.139	0.147	0.137	0.195	0.118	0.155
15	5	1600	2200	41	1	4160	1,400	36,000	0.125	0.157	0.137	0.121	0.159	0.249	0.108
16	3	800	1200	22	11	460	20,000	140,000	0.171	0.129	0.150	0.117	0.199	0.147	0.141
17	9	1200	2000	134	2	100	4,700	57,500	0.281	0.281	0.282	0.282	0.283	0.400	0.439
18	3	800	1200	18	48	2926	20,000	128,000	0.089	0.101	0.096	0.096	0.099	0.144	0.130
19	3	800	1200	23	4	1878	20,000	193,000	0.343	0.367	0.341	0.344	0.358	1.652	1.833
20*	7	2057	2381	28	22	4238	1,000	6,300	0.099	0.097	0.102	0.102	0.095	0.110	0.073
21	5	1600	2100	50	30	903	3,000	36,000	0.152	0.148	0.142	0.148	0.155	0.169	0.201
22*	4	2200	2363	25	12	2519	1,000	9,000	0.104	0.104	0.105	0.107	0.103	0.137	0.214
23*	7	2066	2336	25	40	3576	2,000	11,000	0.110	0.109	0.107	0.109	0.110	0.101	0.093
24*	6	2075	2426	30	22	5720	1,000	10,000	0.223	0.219	0.217	0.213	0.234	0.121	0.092
25*	3	2111	2264	16	105	2971	2,000	7,000	0.113	0.103	0.117	0.109	0.111	0.095	0.105
26	3	1600	2000	14	13	1717	5,000	90,000	0.256	0.207	0.238	0.213	0.281	0.180	0.184
27*	6	2156	2507	30	35	1437	800	6,000	0.074	0.063	0.081	0.069	0.067	0.064	0.074
28	3	1400	1800	16	13	1317	5,000	130,000	0.136	0.134	0.144	0.141	0.128	0.112	0.144
29*	6	2264	2588	45	53	5092	2,000	11,000	0.077	0.074	0.083	0.074	0.078	0.093	0.075
30*	5	2201	2480	34	26	2742	1,500	6,500	0.121	0.114	0.117	0.116	0.120	0.116	0.137
Average of All Short-Time Data									0.184	0.175	0.182	0.173	0.190	0.248	0.277
Rank									4	2	3	1	5	7	6
Average of High-Temperature Data									0.125	0.116	0.125	0.119	0.125	0.137	0.141
Rank									3	1	3	2	3	6	7
Average of High-Stress Data									0.236	0.227	0.232	0.221	0.247	0.345	0.397
Rank									4	2	3	1	5	6	7
Average of Conventional Nickel-Alloy Data									0.179	0.171	0.188	0.175	0.180	0.199	0.253
Rank									3	1	5	2	4	6	7

Table 3 Values of the average deviation of stress and of the average deviation of rupture time

		Average Deviation of Stress							Average Deviation of Rupture-Time						
No.	Rf.	L-M	M-H	S-D	G-S	M-S	C	K	L-M	M-H	S-D	G-S	M-S	C	K
1	12	9.67	8.29	8.90	10.74	11.12	4.40	4.59	99.95	89.75	102.71	95.43	97.47	73.95	74.38
2	13	6.75	6.30	6.07	6.04	7.70	11.90	23.80	73.11	70.55	68.45	67.65	77.68	147.55	179.72
3*	14	10.15	9.71	10.27	10.92	9.64	12.63	12.75	20.35	18.54	20.07	20.92	19.24	29.52	30.62
4	12	8.09	8.23	8.17	8.08	8.39	4.35	4.98	81.80	82.01	82.89	78.79	82.44	54.80	63.52
5*	14	31.49	9.94	45.77	41.74	50.64	9.03	10.09	26.17	21.74	24.90	23.83	24.64	20.29	23.29
6*	14	4.89	2.99	5.97	3.41	4.14	4.70	2.31	20.34	13.35	24.74	14.14	17.65	18.82	10.37
7	15	8.72	5.47	14.08	5.91	7.79	4.80	5.25	82.08	65.89	106.07	64.50	84.56	50.46	82.98
8	15	3.16	3.04	2.86	2.59	3.83	6.51	7.21	33.81	33.15	32.11	30.69	36.30	124.20	154.81
9	16	8.37	8.00	7.56	7.33	9.70	5.47	5.52	73.95	70.80	68.25	64.75	81.62	72.20	80.03
10*	14	13.30	18.45	48.91	14.98	30.89	7.42	6.25	20.85	18.81	22.72	20.40	19.01	33.86	26.83
11*	14	1.26	1.27	0.90	1.03	1.37	2.34	3.25	5.58	3.46	4.12	3.89	6.04	8.85	12.95
12*	14	2.56	1.61	2.18	1.78	2.88	3.40	3.74	10.40	7.77	9.19	7.92	11.55	13.19	15.07
13	16	4.41	4.32	4.10	4.04	4.97	8.85	23.44	48.61	49.31	46.72	45.79	51.82	89.52	174.03
14	17	7.92	5.59	6.58	5.53	9.15	5.38	7.75	31.55	23.10	25.53	22.42	34.54	21.38	29.74
15	18	5.04	6.03	5.50	4.85	6.05	12.85	4.10	24.46	28.80	25.92	23.56	29.28	47.87	19.79
16	19	2.55	1.50	2.15	1.49	3.14	2.79	2.54	27.80	19.89	25.28	18.41	33.16	28.06	26.74
17	16	7.08	7.27	7.43	7.10	7.48	7.26	9.23	65.15	65.31	65.70	65.20	66.13	119.03	145.66
18	15	1.26	1.31	1.33	1.27	1.27	2.29	2.09	13.91	14.05	14.51	13.99	13.31	24.87	23.01
19	15	4.83	5.16	4.88	4.69	5.25	36.31	27.79	66.26	68.76	65.62	63.73	69.53	138.25	902.46
20*	14	3.34	3.16	3.28	3.46	3.12	4.67	2.71	15.09	14.24	15.50	14.99	14.38	20.72	11.99
21	20	5.89	5.15	4.82	5.29	5.99	5.82	6.57	27.45	25.77	24.58	26.16	28.01	33.80	39.95
22*	14	7.50	6.78	7.87	7.18	7.57	6.70	10.32	18.33	17.88	18.51	18.52	18.14	24.07	41.40
23*	14	6.43	6.29	6.25	6.30	6.46	5.06	4.70	18.35	18.85	17.44	17.21	18.25	16.01	15.34
24*	14	8.60	18.06	7.62	7.22	8.95	3.73	3.22	36.24	37.01	36.00	35.77	37.08	19.39	16.14
25*	14	4.22	3.65	4.12	4.02	3.90	3.51	3.56	17.95	15.77	28.09	16.84	17.15	16.27	16.57
26	15	8.63	5.29	7.22	5.85	9.49	5.14	5.29	45.23	31.80	40.35	33.84	49.65	30.46	32.39
27*	14	3.37	2.48	3.64	2.67	2.87	2.73	2.86	13.55	10.32	14.71	11.41	11.99	12.09	12.73
28	15	3.73	3.47	4.09	3.68	3.42	3.56	3.92	22.08	20.52	22.76	22.80	20.64	20.50	25.77
29*	14	4.11	3.92	4.44	3.98	4.20	5.62	4.19	13.96	12.98	15.19	13.06	14.30	18.01	14.18
30*	14	6.97	6.33	6.81	6.43	7.03	6.71	7.35	20.65	19.10	19.82	19.21	20.74	20.99	26.03
Av.		6.81	5.97	8.46	6.65	8.28	6.86	7.38	39.33	32.97	35.95	32.53	36.88	44.97	77.62
Rank		3	1	7	2	6	4	5	5	2	3	1	4	6	7
Av.*		7.73	6.40	11.29	8.22	10.26	5.59	5.52	18.42	16.41	18.64	17.01	17.87	19.43	19.54
Rank		4	3	7	5	6	2	1	4	1	5	2	3	6	7
Av.**		6.01	5.59	5.98	5.28	6.58	7.98	9.00	57.62	47.47	51.09	46.11	53.52	67.31	128.43
Rank		4	2	3	1	5	6	7	5	2	3	1	4	6	7
Av.***		5.54	5.27	5.70	5.01	5.14	6.80	7.71	33.11	30.96	35.06	31.46	33.64	44.69	97.14
Rank		4	3	5	1	2	6	7	3	1	5	2	4	6	7

designed and built under the direction of the senior author, generally involve extremely high test temperatures, with homologous temperatures sometimes being in excess of 0.90; rupture times are generally comparatively short and stress levels are quite low. The other data sets considered more conventional test temperatures and achieved very short rupture times by the utilization of high stresses. The test results of Clough and his associates[17, 18] generally utilized rather high test temperatures, but stress levels span the gap between the two extremes. Korchynsky's very high-temperature data sets have been indicated by asterisks in the left columns of Tables 2 and 3. All other test results will generally be referred to as very high-stress data.

Very short-time, creep-rupture specimens are usually heated to the desired temperatures of test by their inherent resistance to the passage of electrical current (I^2R heating), and there is thus no furnace external to the test specimens. Unusually rapid heating rates, as 1000 °F/sec, are common[18] and sophisticated testing equipment different in nature from that employed for long-time, creep-rupture testing has been developed[17]. Engineers only familiar with conventional testing are prone to criticize testing involving specimen I^2R heating on grounds that temperatures and strains may not be uniform within definite gage lengths which may not be of reduced cross-sectional area. Naturally, such objections have some validity, but it must be realized that the type of testing dealt with in this paper cannot be accomplished in conventional creep-rupture facilities.

Results

Optimum values of the parametric constants which give good correlations, as judged by minimizing the values of the standard deviation of regression, are listed in Table 1. The initials of the considered parametric methods are the column headings (L–M = Larson-Miller, M–H = Manson-Haferd, etc). No effort has been made to give tabular listings for the Conrad and Korchynsky temperature-dependent terms (σ_0, b); these terms are not simple functions of temperature, and for each data set values vary greatly from one set of isothermal test results to another, as was the case with conventional nickel-base alloy results.[1] Table 2 gives values of the standard deviation of regression while Table 3 lists values of the average deviation of stress and average deviation of rupture-time for each individual data set, these values resulting from utilization of the optimum constants of Table 1. Values have been averaged and ranked in the lower horizontal columns, and included are averaged values obtained with the often mentioned nickel-base alloys.[1] In Table 4 the number of times that each parameter gives the best, second best, third best, etc., correlation has been given. In Table 5 the values of the third-order polynomial constants of Eq. (8) have been listed for the cases of the Manson-Haferd and Goldhoff-Sherby parameters.

Results other than those listed in the tables have been obtained with extra computer runs. One objective was to investigate differences in results introduced by utilizing constants other than those with the optimum values of Table 1. Findings with conventional long-time data were verified[1,6] in that it was determined that the fit of data to a master curve, as judged by values of the standard deviation of regression, is relatively sensitive to changes in constant values for those three conventional parameters involving but a single constant. When fitting data by the Manson-Haferd and Goldhoff-Sherby methods two constants are involved and results obtained by use of these parameters are highly insensitive to wide ranges of combinations of constant values; i.e., the error curve is very flat in the vicinity of the minimum. Another objective

Table 4 Number of times with best, 2nd best, 3rd best, etc., correlation

Rank	L-M	M-H	S-D	G-S	M-S	C	K
Standard deviation of regression							
1st	2	6	5	6	3	7	6
2nd	2	7	4	6	2	5	3
3rd	2	10	3	7	1	0	4
4th	11	2	5	7	4	1	0
5th	3	4	7	3	11	1	3
6th	9	0	1	0	4	11	2
7th	1	1	5	1	5	5	12
Av.	4.40	2.83	3.92	2.96	4.66	4.23	4.50
rank	5	1	3	2	7	4	6
Average deviation of stress							
1st	2	4	2	5	2	9	6
2nd	1	8	3	9	2	3	6
3rd	7	8	4	6	0	4	1
4th	7	5	8	4	3	1	2
5th	7	3	4	4	9	1	2
6th	7	1	4	2	6	7	3
7th	1	1	5	0	8	5	10
Av.	4.37	3.07	4.37	2.97	5.17	3.77	4.23
rank	5	2	5	1	7	3	4
Average deviation of rupture-time							
1st	1	9	1	6	1	7	5
2nd	1	7	5	9	3	3	2
3rd	5	7	5	7	3	0	3
4th	11	3	6	5	1	2	2
5th	4	2	4	2	14	1	3
6th	7	1	3	1	3	12	3
7th	1	1	6	0	5	5	12
Av.	4.37	2.63	4.33	2.70	4.77	4.43	4.77
rank	4	1	3	2	6	5	6

of the unreported programs was to investigate effects introduced by modeling master curves by polynomials [Eq. (8)] other than those of third order; the third-order polynomials gave improvements in the values of Tables 2 and 3 as compared with use of second-order polynomials. However, there was no general advantage as regards correlation if polynomial approximations higher than third-order were used.

Discussion and Conclusions

Graphical constructions have been used to approximately verify the value of each parameter constant listed in Table 1. Information obtained from such graphical constructions may be of great importance, particularly as regards the understanding of data behavior, and will quickly determine if a computer run is giving erroneous results. The use of computer programs and statistical evaluations gives specific means for comparisons of results and allows standardization of unbiased techniques. Unfortunately, a single datum point lying well outside of the data trend will have a large influence on computer statistical evaluations, something that can easily be compensated for by graphical means. In view of the sometimes significant differences between values of constants and temperature-dependent terms determined by computer programs and by the often ill-defined graphical methods, it is recommended, when any of the considered parameters are used, that computer evaluations be made in addition to graphical determinations.

There are several points of interest to be discussed in regard to the values of the parametric constants listed in Table 1. For the conventional parameters with but one constant, there are variations of optimum values on using from second- to third-order polynomial models, but the variations are relatively small. With the two conventional parameters involving two constants each, the order of master curve polynomial approximation greatly influences the values of the parameter constants without significantly influencing values of the standard deviation of regression. Over-all or average improvements of 15% in the values of standard deviation of regression are achieved by using third-order polynomial models of master curves instead of second-order models, and the improvements

Table 5 Values of polynomial constants of Eq. (8)

No.	Manson-Haferd c_o	c_1	c_2	c_3	Goldhoff-Sherby c_o	c_1	c_2	c_3
1	3.2910	-2.3900	0.5790	-0.0469	4,400,230	-3,180,056	765,602	-61,522
2	2.0020	-1.4080	0.3264	-0.0253	337,900	-222,301	51,041	-3,921
3	0.0149	-0.0199	0.0075	-0.0010	20,013	-42,046	15,525	-1,947
4	0.2242	-0.1551	0.0350	-0.0027	1,350,455	-881,572	195,203	-14,527
5	0.0305	-0.0371	0.0140	-0.0018	-241,427	196,169	-57,438	5,427
6	0.0501	-0.0333	0.0090	-0.0009	316,935	-217,291	56,948	-5,604
7	0.4460	-0.2676	0.0588	-0.0042	1,434,913	-861,888	178,487	-12,608
8	5.8480	-3.5820	0.7320	-0.0500	1,360,828	-834,063	169,113	-11,460
9	0.7032	-0.5284	0.1322	-0.0111	357,945	-276,770	68,636	-5,717
10	-0.0678	0.0685	-0.0203	0.0020	-867,720	834,561	-249,543	24,210
11	-0.0823	0.0636	-0.0164	0.0013	-75,475	38,929	-100,974	991
12	-0.1901	0.1532	-0.0400	0.0032	-527,943	443,914	-129,084	12,237
13	0.3748	-0.2844	0.0715	-0.0061	157,108	-97,171	24,037	-2,020
14	0.0340	-0.0360	0.0120	-0.0010	-14,732	-6,084	2,998	-499
15	0.0090	-0.0120	0.0030	-0.0003	96,520	-44,230	10,628	-1,032
16	13.8500	-8.9470	1.9270	-0.1384	9,063,128	-5,831,637	1,250,076	-89,404
17	0.1632	-0.1239	0.0306	-0.0026	394,426	-267,695	64,897	-5,359
18	0.6710	-0.4398	0.0950	-0.0069	1,296,779	-785,944	163,349	-11,501
19	1.4000	-0.9761	0.1816	-0.0126	1,953,857	-1,193,358	245,598	-16,895
20	-0.0213	0.0288	-0.0092	0.0009	-299,814	348,678	-112,957	11,532
21	0.0709	-0.0607	0.0169	-0.0016	93,796	-88,381	23,324	-2,155
22	0.4143	-0.3536	0.1023	-0.0099	2,041,340	-1,744,131	505,647	-48,999
23	0.0482	-0.0455	0.0133	-0.0013	72,372	-84,535	24,718	-2,519
24	-0.2520	0.1970	-0.0510	0.0040	-600,819	473,054	-130,271	11,704
25	-2.1770	1.8110	-0.5100	0.0470	-11,594,738	9,944,421	-2,805,809	261,431
26	0.0809	-0.0774	0.0235	-0.0024	-20,958	-8,303	3,947	-664
27	-0.0390	0.0469	-0.0147	0.0015	-457,783	544,421	-180,770	18,683
28	0.0351	-0.0337	0.0079	-0.0006	435,193	-257,792	60,245	-4,983
29	0.0527	-0.0377	0.0104	-0.0010	361,307	-247,788	66,420	-6,431
30	0.0090	-0.0130	0.0050	-0.0010	-33,983	6,574	-824	-156

in values of average deviation of stress and average deviation of rupture-time are even larger. There is no advantage in using polynomial approximations to master curves in excess of the third order.

Larson and Miller[2] suggested that a C value of 20 would possibly be appropriate for all alloys subjected to conventional long-time testing. With the very short-time data the optimum values of the Larson-Miller constant were found to lie between the reasonable limits of 11.94 and 25.53 with an over-all average of 18.36. Established is the fact that there are indeed preferred values of the C constant for optimum correlations, a finding verified by several others when utilizing conventional long-time, stress-rupture test results. Also verified is the fact that the C values for very short-time data lie in the same range as the values resulting from consideration of long-time data. With the very short-time data there are no erroneous C values as those reported by Bendel and Gravenhorst.[21] The values of the Manson-Haferd optimum constants (log t_a and T_a) generally differ greatly from the standard values of 10 and 600, but this finding is not due to peculiarities resulting from the consideration of very short-time data, equivalent findings having been reported by numerous investigators who have worked with conventional long-time test results. The parametric constant ΔH for the Sherby-Dorn method has often been associated with activation energies for solid state diffusion. The over-all average ΔH value of 71,180 cal/mole/°K for ferritic-martensitic alloy steels is somewhat less than the value of 95,880 cal/mole/°K averaged over the long-time data sets for equivalent steels. The average ΔH values of 88,260 cal/mole/°K compares well with an average value of 83,410 cal/mole/°K obtained by considering ten long-time data sets for similar austenitic stainless steels. For the superalloys the average ΔH value of 120,630 cal/mole/°K may be compared with an average value over 31 long-time, nickel-base alloy data sets[1] of 105,650 cal/mole/°K. It is not surprising that there are differences in the ΔH values determined with very short-time data and conventional long-time data, since diffusion can only play a small part in the creep processes when rupture times are measured in seconds or minutes.

The ΔH^* activation energies for the Conrad parameter have extreme values from 6,730–414,820 cal/mole/°K and thus can hardly be related to activation energies for self-diffusion, as claimed by Conrad, although the averaged values for the three classes of considered alloys are 80,400, 93,050, and 64,230 cal/mole/°K, respectively. The mentioned extreme values may be compared with 10,460–219,390 cal/mole/°K for the case of 31 long-time, stress-rupture, nickel-base alloy data sets.[1]

Studies of the computer results listed in Tables 2 and 3 could possibly be interpreted as indicating that the Manson-Haferd, Goldhoff-Sherby, Larson-Miller, Sherby-Dorn, and Manson-Succop parameters may almost equally compete as regards abilities to successfully correlate very short-time, stress-rupture test results. In general, the degree of correlation obtained with the Conrad and Korchynsky parameters are not as attractive as with the other five methods. Further studies of the computer results will show that there are advantages to utilizing either the Manson-Haferd or Goldhoff-Sherby methods. Near the bottoms of Tables 2 and 3 are listed averaged values of the three statistical terms standard deviation of regression, standard deviation of stress, and standard deviation of rupture-time; averages have been made over all of the very short time data sets of Table 1, over Korchynsky's very high-temperature test results marked by asterisks, over the short-time, very high-stress data sets for alloys 1, 2, 4, 7–9, 13–19, 21, 26, and 28, and over the 31 data sets for nickel-base alloys[1] tested over long rupture times by conventional methods. Information listed in the lower averages horizontal columns of Table 2 and 3 indicates that for nearly all cases superior correlations are given by the Manson-Haferd and Goldhoff-Sherby parameters. In Table 4 are listed the number of times which each parameter gives the best, second best, third best, etc, correlation for each of the 30 very short-time data sets, as judged by the three considered statistical terms: there are again strong indications that the Manson-Haferd and Goldhoff-Sherby procedures give superior correlations.

A most important conclusion is that very short-time, stress-rupture test results may be correlated into master curves by utilization of the Manson-Haferd, Goldhoff-Sherby, Larson-Miller, Sherby-Dorn and Manson-Succop methods; superior correlations are obtained by utilization of the two constant Manson-Haferd and Goldhoff-Sherby methods. The degrees of correlation which may be obtained with very short-time data are as adequate as those which may be obtained with conventional long-time, stress-rupture test results, as shown by the averaged values of Table 2 and 3. Utilization of the third-order polynomial constants listed in Table 5 will result in master curves for the 30 considered data sets showing average standard deviation of regression values of 0.175 and 0.173, averages of average deviation of stress values of 5.97 and 6.65, and average values of average deviation of rupture-time of 32.97 and 32.53 for the Manson-Haferd and Goldhoff-Sherby methods, respectively, and it is expected that these values would be approximately applicable for other future data sets obtained with well conducted testing programs. If very short-time, very high-temperature, stress-rupture testing is accomplished, as reported by Korchynsky, improved correlations will be expected to result, as shown by Table 2. The information contained in Table 5 gives the designer data which may be utilized with the same degree of confidence as that resulting from conventional long-time, stress-rupture test programs.

Tables 2 and 3 show that no single parameter is ideally suited for use with all very short-time, stress-rupture data sets. In approaching the treatment of a new material or a new data set a completely open mind should be maintained, and the available data should be used to point of the direction of the most appropriate parameter. Graphical methods are definitely of value at this point, but should be replaced by computer methods at an appropriate early date.

In regard to the values of the optimum parameter constants, there is apparently no connection whatsoever between the magnitudes of constant terms and the metallurgical characteristics or chemical compositions of the involved alloys. There is no over-all dependence of abilities to correlate on the magnitudes of parameter constants from data set to data set; for example, Table 2 shows that data sets 14 and 16 have about the same degree of correlation, but Table 1 shows widely different values for many of the parameter constants for these data sets.

An important conclusion is that the Conrad and Korchynsky methods are very erratic from data set to data set, but the erratic behavior cannot be traced to the values of the parameter constant terms or to the metallurgical characteristics of the involved alloys. Regardless of the erratic behavior, it is nevertheless thought that some special considerations should be given to these parameters since a) they are the only considered parameters involving temperature dependent terms; b) computer programs are original and methods of parameter-constant and temperature-dependent term evaluations have been modified by the authors; c) results obtained with master curves which are linear are being compared with other master curves based on third-order polynomials; d) these are the only parametric terms which have been developed by considerations of dislocation theory; and e) Table 4 shows that the Conrad method actually gives the best correlation the maximum number of times of any of the seven parameters. Unfortunately, it also gives very poor correlations a large number of times. The Conrad method presupposes linearity of isothermal lines on plots of σ against log t_r, whereas it has often been demonstrated that many data sets have increased linearity for isothermal lines on plots of log σ against log t_r. Perhaps a modification of the Conrad method taking this consideration into account would result in a greatly improved parameter.

Conrad and Korchynsky parameters cannot be used to interpolate to temperatures other than those actually involved in the stress-rupture testing program; the temperature dependent terms are irrational functions of temperature.

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A Practical Approach to Spacecraft Structural Dynamics Problems

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This paper is concerned with the system approach for structural dynamics. First, the concept of dynamic mass is presented. Second, the determination of a dynamic mass is given relative to the reaction forces and moments between two substructures in terms of the cantilever normal modes of one substructure. Third, it is shown how one can simply couple and uncouple substructures for dynamics problems. Fourth, an example is given for the digital simulation of a sine wave test for the purpose of calculating rotational response of the shaker. Fifth, forcing functions are mathematically determined from the measured responses of the space vehicle in flight. Finally, it is shown how one can perform a hybrid test simulation by combining a real-time analog computer with digitally determined structural characteristics, shaker properties and actual control equipment in order to investigate the stability of the control loop.

Introduction

Building from the classical normal mode analysis,[1–5] the structural analysis techniques have evolved in several directions in the recent past. The system approach, borrowing from the ideas of electrical engineering, has been introduced in Ref. 6, which has permitted the analytic coupling of substructure analysed at separate times or by separate organizations, as is the case for most aerospace structures. Viewed from a different angle, the system approach has brought the frequency domain into structural dynamics.[7–12] Then, the

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Index categories: Launch Vehicle and Missile Structural Design; Launch Vehicle or Missile System and Component Ground Testing; Unmanned Lunar and Interplanetary System.

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